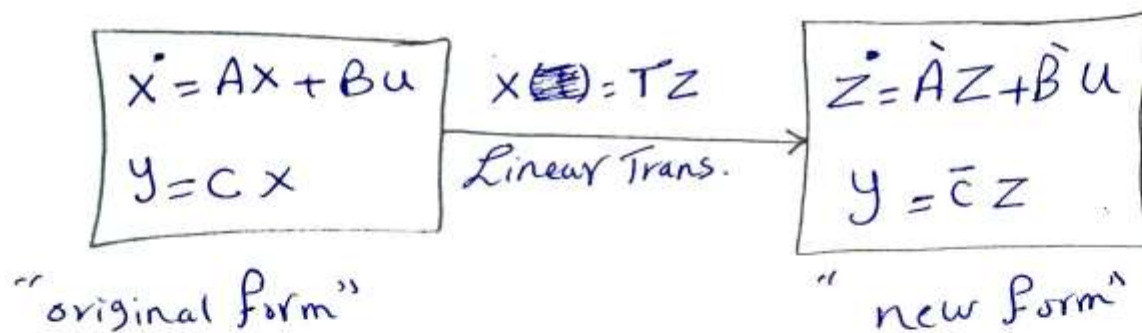


Control Lec 10

→ Transformation from general form of state space to another form :-



← مجرد تحويل رياضي ولكننا نفس ال Poles و zeros و نفس ال T.F

Given: A, B, C

Find $T \Rightarrow$ transformation matrix $\bar{A}, \bar{B}, \bar{C}$ (square) $n \times n$

$$x(t) = T z(t) \quad (\text{Linear transformation})$$

n ← درجة النظام

$$\dot{x} = T \dot{z}$$

$$\dot{x} = Ax + Bu$$

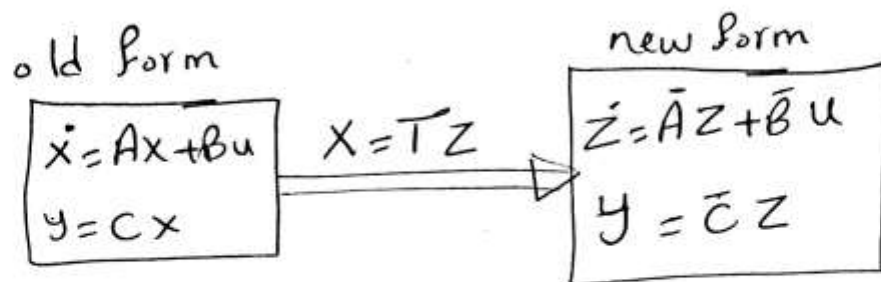
$$T \dot{z} = ATz + Bu$$

$$\dot{z} = T^{-1}ATz + T^{-1}Bu$$

$$\bar{A} = T^{-1}AT$$

$$\bar{B} = T^{-1}B$$

تطلع المعادلات بالشكل ده



→ غالباً ما يتم التحويل من صورة الـ (system) إلى (Diagonal) أو (Controllable)

→ التحويل يتم إلى صورة الـ (Controllable) ثم دراسته.

"Transformation to Diagonal Form"

Assume 2nd order system

$$\bar{A} = T^{-1}AT = \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix}$$

$s_2, s_1 \rightarrow \text{poles}$

$$T = (V_1 \quad V_2), \quad V_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \quad V_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

→ منطلق الـ I بواسطة

①، ② حسب درجة الـ sys.

$$(sI - A)V_i = 0 \quad ; \quad i = 1, 2$$

$$(s_1 I - A)V_1 = 0 \quad ; \quad (s_2 I - A)V_2 = 0$$

$$\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = T^{-1}AT$$

$$T\bar{A} = AT$$

$$T \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = AT$$

$$(V_1 \quad V_2) \begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix} = AT$$

Example

$$\dot{x} = \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} x + \begin{pmatrix} 2 \\ 5 \end{pmatrix} u ; y = (1 \quad 2) x$$

ال chls وال (Poles) كما في الشكل القديم لد (sys) والجديد .

$$\text{Poles} \Rightarrow |sI - A| = 0$$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \right| = 0$$

$$\left| \begin{array}{cc|c} s+5 & (-) 1 & \\ -3 & s+1 & \end{array} \right| = 0 \Rightarrow (s+5)(s+1) + 3 = 0$$

$$s^2 + 6s + 8 = 0$$

\downarrow_i \downarrow_i

$$\text{for } I = 1 \Rightarrow s = -2 \text{ first pole}$$

$$(s_i - A) V_i = 0$$

$$\begin{pmatrix} s_i + 5 & 1 \\ -3 & s_i + 1 \end{pmatrix} v_i = 0$$

$$i=1 \Rightarrow \begin{pmatrix} s_1 + 5 & 1 \\ -3 & s_1 + 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 + 5 & 1 \\ -3 & -2 + 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

نفس المعادلة لكن بالسالب

$$3a_1 + b_1 = 0 \quad ; \quad -3a_1 - b_1 = 0$$

$$a_1 = 1 \quad ; \quad b_1 = -3$$

وبالتالي هنتقرب أول قيمة ب 1

$$\therefore v_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\text{if } i=2 \Rightarrow s_2 = -4 \quad \& \quad v_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$$

$$\begin{pmatrix} s_2 + 5 & 1 \\ -3 & s_2 + 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 \\ -3 & -3 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$$a_2 + b_2 = 0$$

نضرب المعادلة لكه الثانية مفردة في -3

$$-3a_2 - 3b_2 = 0$$

$$\text{assume } a_2 = 1 \Rightarrow b_2 = -1 \Rightarrow V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$T = (V_1 \quad V_2) = \begin{pmatrix} +1 & +1 \\ -3 & -1 \end{pmatrix}$$

$$T^{-1} = \frac{1}{|T|} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix}$$

$$\tilde{A} = T^{-1} A T$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -2 & -4 \\ 6 & 4 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} -1 & -2 \\ 3 & 2 \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} \rightarrow \text{system poles.}$$

□

$$\bar{B} = T^{-1} B$$

$$= \frac{1}{2} \begin{pmatrix} -1 & -1 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -7 \\ 11 \end{pmatrix} = \begin{pmatrix} -3.5 \\ 5.5 \end{pmatrix}$$

$$\bar{C} = C T = (1 \quad 2) \begin{pmatrix} 1 & 1 \\ -3 & -1 \end{pmatrix} = (-5 \quad -1)$$

$$\dot{Z} = \begin{pmatrix} -2 & 0 \\ 0 & -4 \end{pmatrix} Z + \begin{pmatrix} -3.5 \\ 5.5 \end{pmatrix} u$$

نظام (System) معادلة الحالة (state equation) هي

$$y = (-5 \quad -1) Z$$

(Special Case) Transformation Into Diagonal Form

$$\dot{X} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} X + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u$$

$$y = (1 \quad 2) X$$

$$\text{Poles} \Rightarrow \text{ch/s eqn} \Rightarrow |sI - A| = 0$$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ -2 & -3 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix} = 0 \Rightarrow s^2 + 3s + 2 = 0 \rightarrow \text{ch. eqn}$$

$$(s+1)(s+2) = 0$$

Thus Poles $\Rightarrow -1, -2$

$$(sI - A)V_i = 0$$

For $i=1$, $s_1 = -1$, $V_i = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$

$$\begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} = 0$$

$$-a_1 - b_1 = 0 \quad , \quad 2a_1 + 2b_1 = 0$$

assume $a_1 = 1 \Rightarrow b_1 = -1$. $\therefore V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

For $i=2$, $s_2 = -2$, $V_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$

$$\begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} = 0$$

$$-2a_2 - b_2 = 0$$

$$2a_2 + b_2 = 0$$

$$\text{assume } a_2 = 1 \Rightarrow b_2 = -2$$

$$\therefore V_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$T = (V_1, V_2) = \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}$$

$$\bar{A} = T^{-1}AT = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

$$B = T^{-1}B = \begin{pmatrix} 2 \\ -1 \end{pmatrix} ; \bar{C} = CT = (-1 \quad 3)$$

لأن (matrix) في (Controllable) وعاليز تحولها (Diagonal) لها حالة خاصة.

(Special Case) A is Controllable.

$$T = \begin{pmatrix} 1 & 1 \\ s_1 & s_2 \end{pmatrix} ; s_1, s_2 \rightarrow \text{Poles}$$

$$T = \begin{pmatrix} 1 & 1 & 1 \\ s_1 & s_2 & s_3 \\ s_1^2 & s_2^2 & s_3^2 \end{pmatrix} ; s_1, s_2, s_3 \Rightarrow \text{Poles}$$

For Repeated Poles

نترك العمود الأول كما هو ، والعمود الثاني هو تناهض ~~العمود الأول~~ .

Poles $\rightarrow s_1, s_1, s_2$

$$T = \begin{pmatrix} 1 & 1 & 1 \\ s_1 & s_1 & s_2 \\ s_1^2 & s_1^2 & s_2^2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ s_1 & 1 & s_2 \\ s_1^2 & 2s_1 & s_2^2 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 \\ s_1 & 1 \end{pmatrix}$$

$s_1, s_1 \rightarrow \text{Poles}$

هذا ال (Transform) من معادلات الإمتحان

* state feed back control / Pole Placement design
classifications :-

- 1) classical control (PI, PD, PID, Phase(lead, lag))
- 2) modern control (observer, state feedback)
- 3) Computational Artificial Control Design
(neural, fuzzy, ...)

* modern Control design

→ The system model based on s.s (state space)

جقق الإمتطام قبل كده

Control

1) servo Problem
control / design

I/P $\neq 0$ ($r \neq 0$)

regulation Problem
design / Control

($r=0$) → I/P Zero

noise , error (system) في حالة
كبيرة عايز يتلاشها فبالتا لي يفحص وجود o/p
لد Sys. دونه وجود I/P
(state feed back control) حالة منها

* Regulation Problem design :-

when control is done to reject (desired I/P),
The o/p match this setting point, this
Process is called servo control.

* state Feedback Control

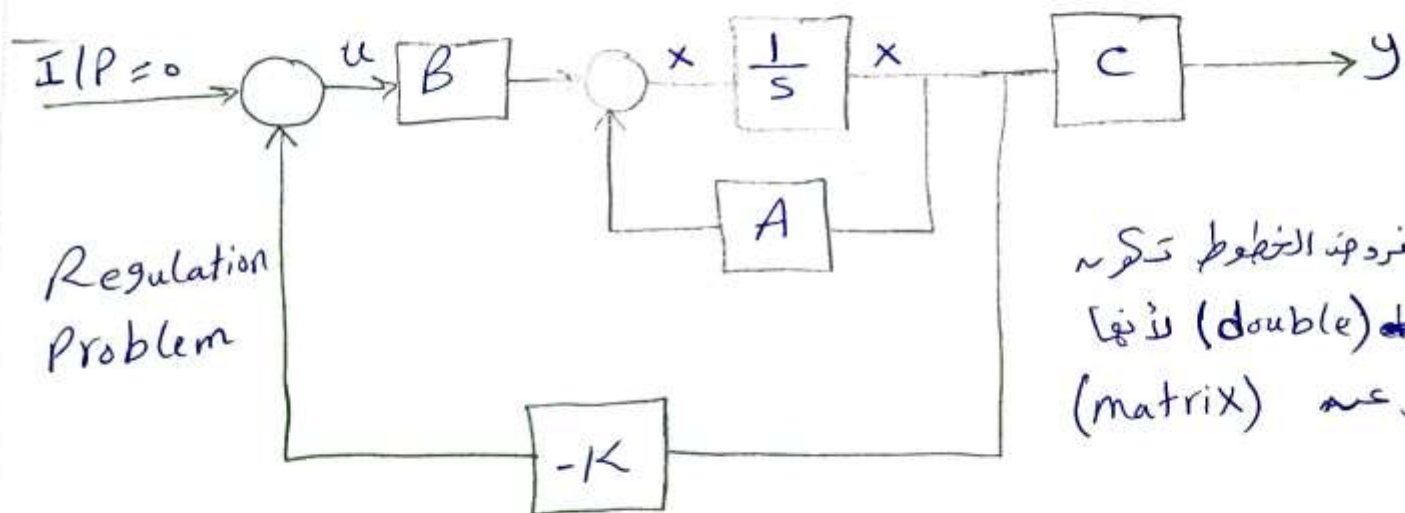
- regulation problem
- modern control design.

Given
$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

Control Law :- $u = -Kx$
↳ Gain matrix

← بالشكل ده مفيش I/P واعتبرت اى (Disturbance) اللى قدر طلع o/p .

→ The design is to find the Gain K



← المفرد هالتخطوط كده ~
(double) لأنها
تعتبر عنه (matrix)

* To Find the Gain K

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$u = -Kx$$



$$\dot{x} = Ax - BKx$$

$$= (A - BK)x$$

state eqn after using بالقوة
control loop

The desired chls equation

$$|sI - (A - BK)| = 0 \Rightarrow (1)$$

* Given design specs

The location of the desired poles

Assume 2nd order system

Given: p_1, p_2 (poles)

The desired chls eqn $\alpha_c(s)$

$$\alpha_c(s) = (s - p_1)(s - p_2) = 0 \rightarrow (2)$$

From (1), (2) we can obtain the Gain K

$$K = [K_1, K_2, \dots, K_n]$$

$n \rightarrow$ system order.

Example $\dot{x} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t)$

$$y = \begin{pmatrix} 1 & -1 \end{pmatrix} x$$

* Design a state feedback controller to place (locate) the closed loop poles at $s = -1 \pm j2$

① Desired chls eqn $\alpha_c(s)$:-

$$\alpha_c(s) = \underbrace{\left(\underbrace{s+1}_{x} - \underbrace{j2}_{y} \right)}_{x} \underbrace{\left(\underbrace{s+1}_{x} + \underbrace{j2}_{y} \right)}_{y} = 0$$

$$\alpha_c(s) = (s-1)^2 + 4 = 0$$

$$\alpha_c(s) = s^2 - 2s + 5 = 0 \longrightarrow \textcircled{1}$$

② The desired chls eqn After using Control Law $u = -Kx$

$$|sI - (A - BK)| = 0$$

$$A - BK = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} K_1 & K_2 \end{pmatrix}$$

$$A - BK = \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} K_1 & K_2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 - K_1 & -K_2 \\ 1 & 0 \end{pmatrix}$$

$$|sI - (A - BK)| = 0$$

$$\left| \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 2 - K_1 & -K_2 \\ 1 & 0 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} s - 2 + K_1 & K_2 \\ -1 & s \end{vmatrix} = 0$$

$$s^2 - 2s + K_1s + K_2 = 0$$

$$\cancel{s^2 + (2 + K_1)s + K_2} = s^2 + (K_1 - 2)s + K_2 = 0 \Rightarrow (2)$$

ch.s
desired
eqn.

← بمقارنة المعاملات بعد ذلك رقم (1)

$$s^2 - 2s + 5 = 0$$

$$K_1 - 2 = 2 \Rightarrow K_1 = 4 \quad \& \quad K_2 = 5$$

قلم

"check controllability"

$$M_c = (B \quad AB)$$

لا (system) لازم يبقى (Controllable)

عشان تعمل ال (design).

$$|M_c| \neq 0 \text{ Controllable}$$

لازم تعمل (check) قبل الحل.

← ممكن يجي في الامتحان.

Another solution

* Ackermann's method

Replace $s \rightarrow A$

$$K = [K_1 \ K_2] = [0 \ 1] M_c^{-1} \alpha_c(A)$$

$$\alpha_c(A) \Rightarrow \alpha_c(s) \Big|_{s=A} \quad \text{Desired ch. eqn}$$

$M_c \Rightarrow$ Controllability matrix

* In 3rd order

$$K = [K_1 \ K_2 \ K_3] = (0 \ 0 \ 1) M_c^{-1} \alpha_c(A)$$

1) check controllability

$$|M_c| \neq 0 \quad M_c = (B \ AB)$$

$$M_c = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow M_c^{-1} = \begin{bmatrix} 1 & 2*-1 \\ -1*0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$\therefore |M_c| = 1 \neq 0 \quad (\text{Controllable system})$$

$$K = (0 \quad 1) M_c^{-1} \alpha_c(A)$$

$$M_c^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

(حسابها به مشورت)

$$\alpha_c(s) = s^2 + 2s + 5 = 0$$

$$\alpha_c(A) = \alpha_c(s) \Big|_{s \rightarrow A} = A^2 + 2A + 5I$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 5 & 0 \end{pmatrix}$$

$$K = (0 \quad 1) \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 13 & 0 \\ 4 & 5 \end{pmatrix}$$

$$= (0 \quad 1) \begin{pmatrix} 13 & 0 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ \downarrow K_1 & \downarrow K_2 \end{pmatrix}$$

Report

$$\dot{x} = \begin{pmatrix} -1 & 0 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u$$

$$y = (1 \quad 0) x$$

~~Determine the feedback~~
gain

Determine the feedback gain K to make

$$\zeta = 0.5 \text{ \& \& } t_s = 2 \text{ sec}$$

$$t_s = \frac{4}{\zeta \omega_n} \text{ calculate } \omega_n \text{ then}$$

$$s_{1,2} = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$$

$$\zeta \omega_n = 2 \Rightarrow \omega_n = 4$$

$$s_{1,2} = -2 \pm j 3.46$$

Final
Exam



Root locus

Bode Diagram

Design lead & lag

check Controllability

Example

مطابق الیوم